Parametrization and validation of a nonsmooth discrete element method for simulating flows of mineral ore balls

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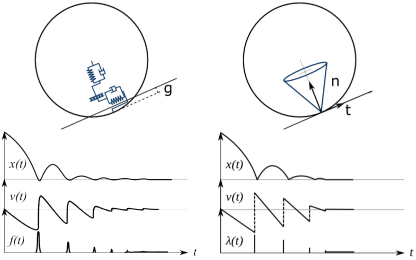
Highlights

- Developing nonsmooth DEM methods for fast simulation
- Application to design exploration of pellet drum outlet
- New: Material characterization and validation
- New: Constraint based rolling resistance

Application: geometric design exploration

- Iron ore balling circuit
- Simulation based design optimization
- Previous research: **"Outlet design optimization based on large-scale nonsmooth DEM simulation, D. Wang (2013)"**
- Speed: 220s per 1s, 2000 designs run on 12 cores for 1 week

Smooth versus nonsmooth DEM



Smooth DEM (SDEM)	Nonsmooth DEM (NDEM)
smooth penalty forces	constraints & impulses
small time-step	large time-step
well proven	solver & state dependent

Example: time step $h_{\mathrm{SDEM}} = 10^{-6}$ s and $h_{\mathrm{NDEM}} = 10^{-3}$ s
for $d = 10$ mm, $\rho = 2500$ kg/m^3, $Y = 10$ GPa, $u_n \lesssim 0.1$ m/s (Servin, 2013).

Nonsmooth Discrete Element Method (NDEM)

Multibody system (x, v), $v = dx/dt$ & constraint force $G^T\lambda$

$$M\frac{dv}{dt} = -\nabla U(x) + F(x, v, t) + G^T\lambda \qquad (1)$$

$$(x^i, v^i) \rightarrow \begin{cases} (x^{i+1}, v^{i+1}, \lambda^{i+1}) \\ \mathrm{law}_{SC}[v^{i+1}, \lambda^{i+1}] = \mathrm{true} \end{cases} \qquad (2)$$

Signorini-Coulomb law & Newton impact law

$$\mathbf{f}_n \geqslant 0, \quad \mathbf{u}_n \geqslant 0, \quad \mathbf{f}_n \cdot \mathbf{u}_n = 0, \quad \mathbf{u}_n^+ = -e\mathbf{u}_n^-$$

$$\mu|\mathbf{f}_n| \geqslant |\mathbf{f}_t|, \quad |\mathbf{u}_t|(\mu|\mathbf{f}_n| - |\mathbf{f}_t|), \quad \mathbf{f}_t^T\mathbf{u}_t = -|\mathbf{f}_t||\mathbf{u}_t| \qquad (3)$$

References: Moreau (1988), Jean (1999), Radjai (2009), Lacoursière (2007), Servin (2013)

Mixed linear complementarity problem (MLCP)

$$Hz + b = w_+ - w_-$$
$$0 \leqslant z - l \perp w_+ \geqslant 0$$
$$0 \leqslant u - z \perp w_- \geqslant 0$$

$$H = \begin{bmatrix} M & -\bar{G}_t^T & -\bar{G}_n^T \\ \bar{G}_t & \Gamma & 0 \\ \bar{G}_n & 0 & \Sigma \end{bmatrix}, \quad z = \begin{bmatrix} v^{i+1} \\ \lambda_t^{i+1} \\ \lambda_n^{i+1} \end{bmatrix}, b = \begin{bmatrix} -Mv_{free} \\ 0 \\ \frac{4}{h}\Upsilon\bar{g} - \Upsilon\bar{G}_n v_i \end{bmatrix}$$

Constraint regularization mapping to Hertz model

$$\bar{g} = g^{\alpha} \quad , \quad \bar{G}_n = \alpha g^{\alpha-1}[n, -n] \quad , \quad \alpha = 5/4$$

$$\Gamma = \frac{\gamma_t}{h}, \; \Sigma = \frac{4}{h^2}\frac{\varepsilon_n}{1+4\frac{\tau_n}{h}}, \; \Upsilon = \frac{1}{1+4\frac{\tau_n}{h}}, \; \varepsilon_n^{-1} = \alpha k_n, \; \gamma_n^{-1} = k_n c/\alpha$$

In quasi-static regime:

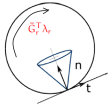
$$\bar{G}^T\lambda \to \underbrace{k_n\left[g^{3/2} + cg^{1/2}\dot{g}\right]}_{f_n} n + \underbrace{\text{proj}_{\mu|f_n|}\left(-k_t v_t^i\right)}_{f_t} t$$

Constraint based rolling resistance

Model:

- Constraint: $\bar{G}_r v = 0$
- Constraint force: $\bar{G}_r^T \lambda_r$
- Constraint force limit: $|\bar{G}_r^T \lambda_r| \leqslant \mu_r r \bar{G}_n^T \lambda_n$

$$H = \begin{bmatrix} M & -\bar{G}_c^T & -\bar{G}_r^T \\ \bar{G}_c & \Sigma_c & 0 \\ \bar{G}_r & 0 & \Sigma_r \end{bmatrix}, \quad z = \begin{bmatrix} v^{i+1} \\ \lambda_c^{i+1} \\ \lambda_r^{i+1} \end{bmatrix},$$



Observe:

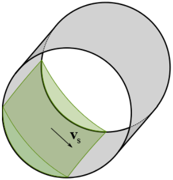
- No explicit velocity dependency
- Particle resting on inclined plane

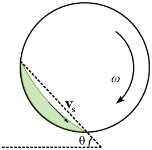
Characterization of material

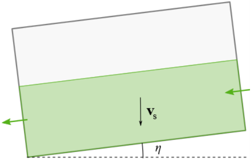
- Laboratory green pellets
- Mass and geometry:
  13 ± 1mm sphere
- Elasticity: 6 ± 0.7 MPa
- Restitution: 0.18 ± 0.04
- Friction: 0.7
- Rolling resistance:
  0.30 ± 0.02

Validation systems

- Pile formation
- Flow profile in drum
- Conveyor profile

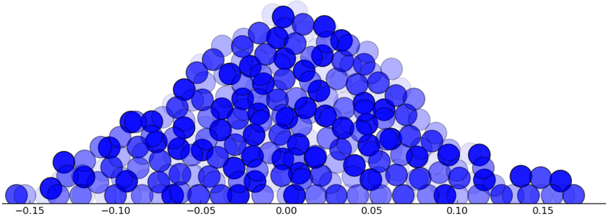






Validation: pile formation

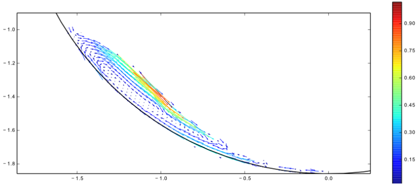
- Measured angle of repose: $35^\circ \pm 5^\circ$
- Simulated angle of repose: 40°

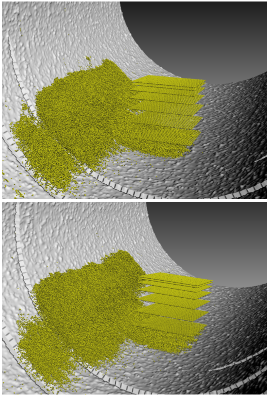


Validation: flow profile in drum

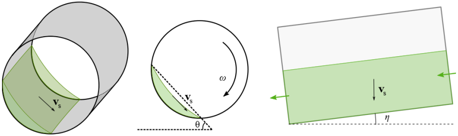
Specification:

- Drum: 4 m
- Rotation speed: 0.25 rad/s
- Input flow: 500 ton/h
- Inclination angle: 7°
- "Texture": 700 dimples/m^2





Validation: flow profile in drum



Measured profile:

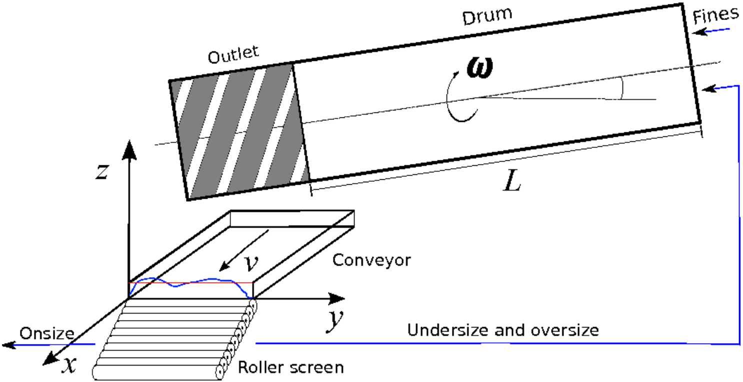
- Charge angle: $\theta = 35°$
- Surface velocity: 0.65 m/s
- Axial flow velocity: 0.2 m/s

Simulated profile:

- Charge angle: $\theta = 33°$
- Surface velocity: 0.5 m/s
- Axial flow velocity: 0.3 m/s

Validation: conveyor bed profile

- Simulation setup



Validation: conveyor bed profile

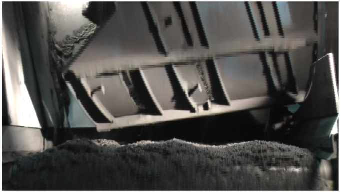
In progress !

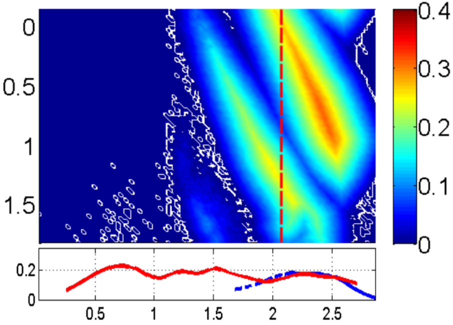
Measured bed profile:

- Video
- Profile

Simulated bed profile:

- Surface
- Profile



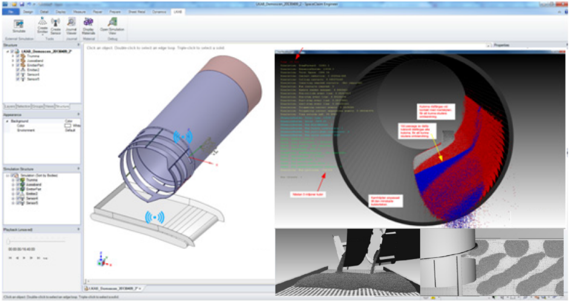


Conclusion

- Material properties
- Constraint based rolling resistance
- Validation of pile formation
- Flow of drum - follow up
- Conveyor bed profile - on going.

Future development

- Parallelization of NDEM PGS show good speed-up
- Simulation and CAD integration → **New tool!**
  - SpaceClaim Motion Dynamics using AgX Multiphysics



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