Regularized multibody dynamics with dry frictional contacts

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The computation of contact forces for non-penetrating bodies subject to dry friction is particularly difficult. There is not even agreement on the correct analytic formulation of dry frictional contacts laws [1]. Our work in this field focuses on real-time simulation of heavy machines such as cranes, track and wheel loaders among others, for the purpose of operator training. Dry friction plays a central role in these, and contact forces are strongly coupled with stiff multibody systems. Performance and stability of the numerical methods are paramount. Direct solvers are often necessary because of the mass ratios involved and the need for fidelity. These are not scalable like iterative ones, and do not allow simple and intuitive implementations such as non-linear Gauss-Seidel relaxation methods. A unified framework including all aspects of the dynamics at the theoretical level, the time discretization, and the numerical solvers themselves is required, and this is what we present.

Consider the Lagrangian $\mathcal{L}(q, \dot{q})$ for a finite dimensional system with generalized coordinates q, as well as indicator functions $g(q, t), a(q, \dot{q}, t)$ for holonomic and non-holonomic constraints, respectively. Our technique is to augment the Lagrangian and introduce the dissipation potential as follows

$$
\begin{aligned}
\mathcal{L}(q, \dot{q}) + \frac{\epsilon}{2} \lambda^T \lambda + \lambda^T g \\
\mathfrak{R} = \frac{\epsilon \tau}{2} \dot{\lambda}^T \dot{\lambda} + \tau \dot{\lambda}^T G \dot{q} + \frac{\gamma}{2} \dot{\alpha}^T \dot{\alpha} + \dot{\alpha}^T a.
\end{aligned}
\tag{1}
$$

The parameters $\epsilon, \gamma \geq 0$ are singular perturbation which yield perfect constraints as $\epsilon, \gamma \downarrow 0$, including $(\partial g / \partial q) \dot{q} = 0$. The parameter $\tau \geq 0$ is a time constant for dissipation and produces constraint stabilization needed in numerical integration.

This Lagrangian is then seen as that of the two sets of bodies, namely, those with generalized coordinates q and strictly positive kinetic energy, and those with generalized coordinates λ with zero kinetic energy, the ghosts. The equations of motion of the system are then the standard Euler-Lagrange equations for constrained multibodies in descriptor form, and subject to the non-potential forces $-\partial\mathfrak{R}/\partial\dot{q}$, $-\partial\mathfrak{R}/\partial\dot{\lambda}$, and $-\partial\mathfrak{R}/\partial\dot{\alpha}$ acting on the q, λ, and α variables, respectively. Inequality constraints lead to variational inequalities as equations of motion. Note that restrictions $c(q) \geq 0$ are never relaxed in the present framework. The ghost velocities $\dot{\alpha}$ and $\dot{\lambda}$ are novel elements providing for a unified treatment of holonomic and non-holonomic constraints.

Considering the variables λ as true mechanical degrees of freedom of ghost particles, it is then possible to impose constraints and potentials of dissipation on these by introducing secondary ghosts. When this is done, we can construct a Lagrangian and potential of dissipation of the form

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}(q, \dot{q}) + \frac{\epsilon}{2}\nu^T c(q) \\
\mathfrak{R} &= (\gamma/2)\dot{\beta}^T\dot{\beta} + \dot{\beta}^T D(q)\dot{q} + (\delta/2)\dot{\sigma}^T\dot{\sigma} + \dot{\sigma}^T(\mu\nu - \|\dot{\beta}\|) + \mathfrak{R}(q, \dot{q}).
\end{aligned}
\tag{2}
$$

Here, $\mathcal{L}(q, \dot{q})$ is the Lagrangian corresponding to a given multibody system with generalized variables q, and subject to potentials of dissipation $\mathfrak{R}(q, \dot{q})$, which may include ghosts. The vector $\nu \geq 0$ contains the normal forces corresponding to a non-penetration constraint with vector indicator $c(q) \geq 0$, $\dot{\beta}$ are the contact tangent forces, $D(q)\dot{q}$ is the projection of the velocities in the tangential contact plane, and $\dot{\sigma} \geq 0$ is the tangential contact speed.

With these definition, the variational inequalities of motion are a solvable mixed non-linear complementarity problem by applying the Euler-Lagrange equations to each of the variables, q, ν, β and σ. Other constitutive laws for viscoplastic materials and a variety of non-ideal constraints can be constructed with the same approach.

We then construct an integrator using discrete time variational mechanics [2]. The resulting semi-implicit method is stable in the limit of vanishing perturbation. A suitable linearization yields robust constraint stabilization.

Finite regularization corresponds directly to physical elasticity and viscosity parameters, and the equivalence can be verified numerically. On the computational side, the regularization parameters correspond directly to smoothing used in interior point and smoothed Newton methods for solving complementarity problems for instance. Using finite values for the perturbations avoids degeneracy and ill-conditioning and can considerably accelerate the convergence of iterative methods such as the common non-linear Gauss-Seidel ones, as we show with simulation data.

References

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