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Validation of a nonsmooth DEM by confined compression and rod penetration

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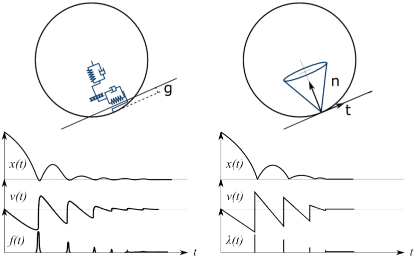
Highlights

- Nonsmooth DEM allows fast simulations
- Does it also produce correct physics?
- Validation by compression and rod penetration experiments
- Agree well with experiments and smooth DEM

In collaboration with: Algoryx Simulation and LKAB

Smooth versus nonsmooth DEM



Smooth DEM (SDEM)	Nonsmooth DEM (NDEM)
smooth penalty forces	constraints & impulses
small time-step	large time-step
well proven	solver & state dependent

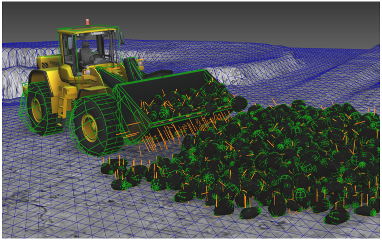
Nonsmooth DEM also known as Contact Dynamics (Moreau, 1988)

Nonsmooth DEM - example

Realtime visual interactive 3D simulation

- articulated vehicle
- driveline
- granular material
- operator-in-the-loop
- time-step 20 ms (60 Hz)
- $20 + 1000$ bodies
- 10^4 variable MLCP
- direct-iterative split solver

Reference: Lacoursière (2013)



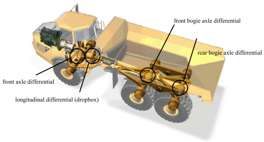
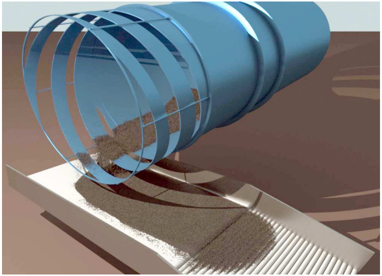


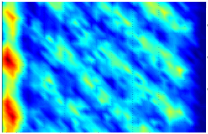
Image courtesy: Algoryx, Oryx, Volvo CE

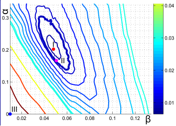
Nonsmooth DEM - example

Simulation-based design of balling drum (LKAB)

- 1K – 1M granules
- 10K – 10M MLCP
- 0.7 – 3.7 m drum diameter
- time-step 1 – 10 ms
- 100/1 of realtime for 1M
- 2D design space 1 week







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Nonsmooth Discrete Element Method (NDEM)

Multibody system (x, v), $v = dx/dt$ & constraint force $G^T\lambda$

$$M\frac{dv}{dt} = -\nabla U(x) + F(x, v, t) + G^T\lambda \tag{1}$$

$$(x^i, v^i) \rightarrow \begin{cases} (x^{i+1}, v^{i+1}, \lambda^{i+1}) \\ \text{laws}_{SC}[v^{i+1}, \lambda^{i+1}] = \text{true} \end{cases} \tag{2}$$

Signorini-Coulomb law Newton impact law

$$\mathbf{f}_n \geqslant 0, \quad \mathbf{u}_n \geqslant 0, \quad \mathbf{f}_n \cdot \mathbf{u}_n = 0$$

$$\mu|\mathbf{f}_n| \geqslant |\mathbf{f}_t|, \quad |\mathbf{u}_t|(\mu|\mathbf{f}_n| - |\mathbf{f}_t|), \quad \mathbf{f}_t^T\mathbf{u}_t = -|\mathbf{f}_t||\mathbf{u}_t| \tag{3}$$

Newton impact law $\mathbf{u}_n^+ = -e\mathbf{u}_n^-$

References: Moreau (1988), Jean (1999), Radjai (2009), Lacoursière (2007), Servin (2013)

Mixed linear complementarity problem (MLCP)

$$\begin{aligned} Hz + b &= w_+ - w_- \\ 0 \leqslant z - l &\perp w_+ \geqslant 0 \\ 0 \leqslant u - z &\perp w_- \geqslant 0 \end{aligned} \tag{4}$$

$$H = \begin{bmatrix} M & -\bar{G}_t^T & -\bar{G}_n^T \\ \bar{G}_t & \Gamma & 0 \\ \bar{G}_n & 0 & \Sigma \end{bmatrix} \; , \; z = \begin{bmatrix} v^{i+1} \\ \lambda_t^{i+1} \\ \lambda_n^{i+1} \end{bmatrix} , \; b = \begin{bmatrix} -Mv_{free} \\ 0 \\ \frac{4}{h}\Upsilon\bar{g} - \Upsilon\bar{G}_n v_i \end{bmatrix}$$

Constraint regularization mapping to Hertz model (Servin, 2013)

$$\bar{g} = g^{\alpha} \quad , \quad \bar{G}_n = \alpha g^{\alpha-1}[n, -n] \quad , \quad \alpha = 5/4$$

$$\Gamma = \frac{\gamma_t}{h} \; , \; \Sigma = \frac{4}{h^2}\frac{\varepsilon_n}{1+4\frac{\tau_n}{h}} \; , \; \Upsilon = \frac{1}{1+4\frac{\tau_n}{h}} , \; \varepsilon_n^{-1} = \alpha k_n \; , \; \gamma_n^{-1} = k_n c/\alpha$$

$$\bar{G}^T\lambda \rightarrow \underbrace{k_n\left[g^{3/2} + c g^{1/2}\dot{g}\right]}_{f_n} n + \underbrace{\mathrm{proj}_{\mu|f_n|}\left(-k_t v_t^i\right)}_{f_t} t$$

Nonsmooth DEM - numerical solver

Direct MLCP solver

- block pivoting method
- block sparse LDLT factorization
- fill-reducing reordering
- BLAS3 optimized

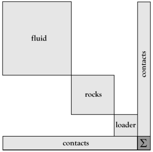
Iterative MLCP solver

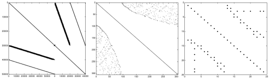
- block sparse projected Gauss-Seidel (pGS)

Hybrid direct-iterative split solver

- vehicle vs granular
- normals vs tangents

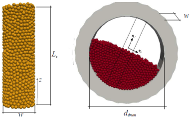
$$\begin{bmatrix} \boldsymbol{M} & -\boldsymbol{G}_A^T & -\boldsymbol{G}_B^T & -\boldsymbol{G}_C^T \\ \boldsymbol{G}_A & \boldsymbol{\Sigma}_A & 0 & 0 \\ \boldsymbol{G}_B & 0 & \boldsymbol{\Sigma}_B & 0 \\ \boldsymbol{G}_C & 0 & 0 & \boldsymbol{\Sigma}_C \end{bmatrix} \begin{pmatrix} \boldsymbol{v} \\ \boldsymbol{\lambda}_A \\ \boldsymbol{\lambda}_B \\ \boldsymbol{\lambda}_C \end{pmatrix} = \begin{pmatrix} \boldsymbol{b}_v \\ \boldsymbol{b}_A \\ \boldsymbol{b}_B \\ \boldsymbol{b}_C \end{pmatrix}$$

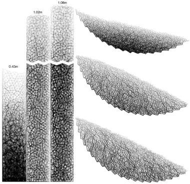




Reference: Lacoursière (2013)

Comparison SDEM & NDEM PGS

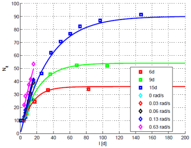




$$
\begin{aligned}
h_{\text{SDEM}} &\gtrsim \sqrt{m/k} \\
t_{\text{SDEM}} &= \sqrt{\frac{k}{m}} K_{\text{SDEM}} N_p t_{\text{real}}
\end{aligned}
$$

$$
\begin{aligned}
h_{\text{NDEM}} &\gtrsim \min(\epsilon d/v_n, \sqrt{2\epsilon d/g}) \\
t_{\text{NDEM}} &= \frac{K^{\text{GS}}_{\text{NDEM}} N_{\text{it}} N_p}{\min(\epsilon d/v_n, \sqrt{2\epsilon d/g})} t_{\text{real}}
\end{aligned}
$$

Number of PGS iterations



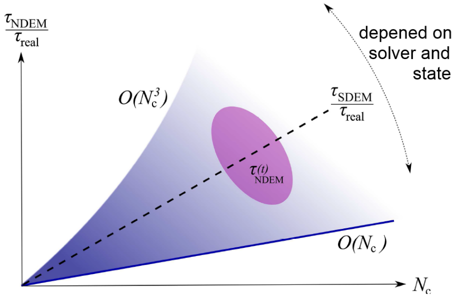
$$
N^{\epsilon}_{\text{it}} = \frac{c_0(1+c_1 I)}{\epsilon} \begin{cases} w(1 - \exp[-\frac{c_2 l}{w}]) & , w \gtrsim 5 \\ c_2 l & , w < 5 \end{cases}
$$

Error tolerance ϵ, inertial number I, coefficients $c_i \sim 1$. Width w, height l.
$K_{\text{SDEM}} = K^{\text{GS}}_{\text{NDEM}} = 10^{-6}$ s.

Comparison SDEM & NDEM PGS

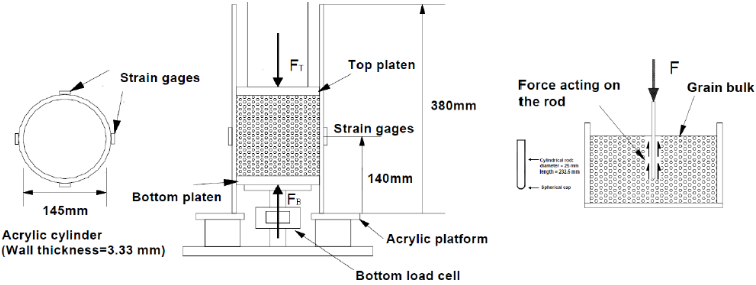
Scaling ratio (Servin at al, 2013) `http://umit.cs.umu.se/granular/dem/`

$$\frac{\tau_{\mathrm{NDEM}}}{\tau_{\mathrm{SDEM}}} = \sqrt{\frac{\max\left(\frac{mv_n^2}{\epsilon^2}, \frac{2mgd}{\epsilon^1}\right)}{kd^2}} \frac{K^{GS}_{NDEM}}{K_{\mathrm{SDEM}}} \frac{N_c}{N_p} \frac{c_0(1+c_1 I)}{\epsilon} \cdot \begin{cases} w(1-e^{-\frac{c_2 l}{w}}) & w \gtrsim 5 \\ c_2 l & w < 5 \end{cases}$$



Experiment setup

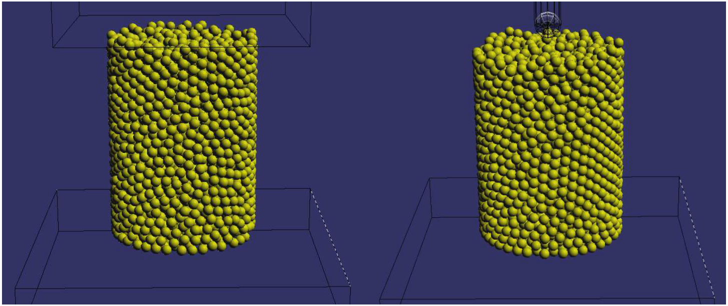
Y.C. Chung and J.Y. Ooi, Confined compression and rod penetration of a dense granular medium: discrete element modelling and validation, in Modern Trends in Geomechanics, eds Wu and Yu, Springer (2006).



3591 glass beads, 10 mm, elasticity $Y = 41$ GPa, friction 0.24, restitution 0.79, density 2530 kg/m^3.
Piston and rod velocity: 50 mm/min. Load force up to 1000 N.
SDEM simulations run with EDEM (Chung & Ooi, 2006). Performance measured with LIGGGHTS.

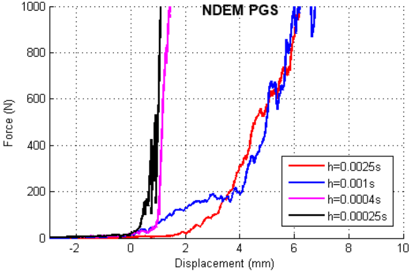
Simulation setup - NDEM

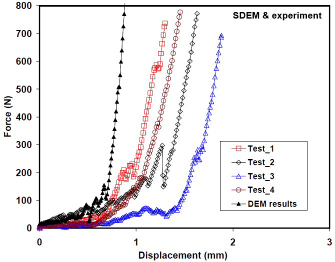
- Elasticity: $Y \to \epsilon_n$ regularization in MLCP
- Damping time: $\tau = 2h$
- Time-step: $h = 0.25 - 4$ ms (smooth DEM 0.001 ms)
- Iterative solver: projected Gauss-Seidel, 500 iterations
- Direct solver: direct-iterative block pivot



Results - effective stiffness in compression

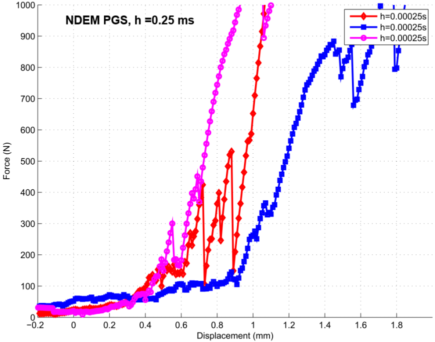
	NDEM PGS	NDEM dir	SDEM	exp
k [N/m]	$2-3$	2	3	2
h [ms]	$0.25-0.4$	4	0.001	
τ_X/τ	10	1000	1000	

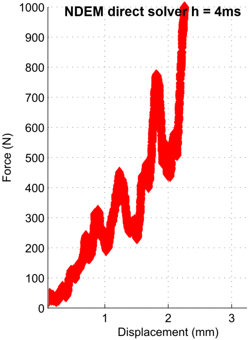




Results - effective stiffness in compression

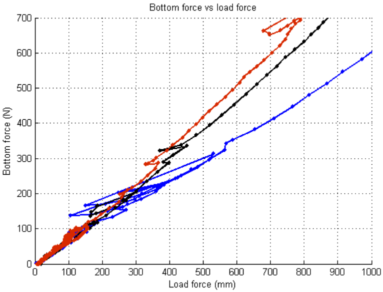
Samples

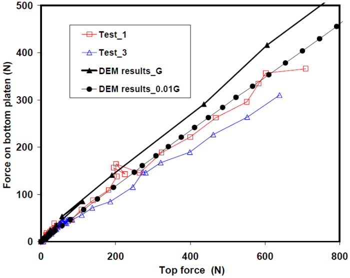




Results - load vs bottom force in compression

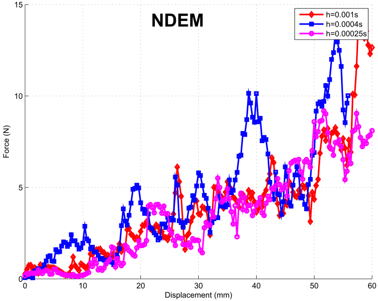
	NDEM PGS	SDEM	exp
F_b/F_l	0.6 – 0.8	0.7	0.6
h [ms]	0.25	0.001	

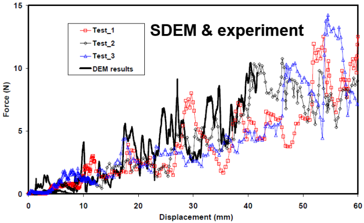




Results - rod penetration

	NDEM PGS	SDEM	exp
k [N/m]	0.17	0.17	0.17
h [ms]	0.25	0.001	





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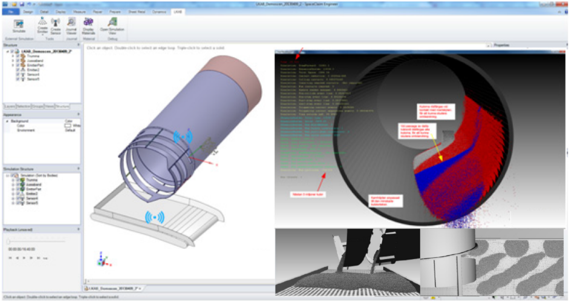
Summary and conclusions

- NDEM with constraint based regularization from Hertz law has been validated
- Agree well with experiment & SDEM
- Next: quantify and understand deviation
- PGS solver required time-step $h \lesssim 0.4$ ms
- NDEM allows large time-step for compression tests: $\lesssim 0.4$ ms PGS, $\lesssim 4$ ms direct, $\lesssim 0.001$ ms SDEM

Future development

- Constraint based rolling resistance (ThA03, D. Wang)
- Parallelization of NDEM PGS show good speed-up
- Simulation and CAD integration → **New tool!**
  - *Dynamics for SpaceClaim* using AgX Multiphysics
- Strong & weak coupling with other tools via FMI



algoryx MULTIPHYSICS AND 3D SIMULATION SPACECLAIM CORPORATION LKAB Optimation

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